

Tidal damping in a shallow estuary: a one-dimensional model

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Abstract

We fit a one-dimensional tidal model to water-level records from six stations along a 42 km estuary. The model reproduces the observed decay of the semidiurnal amplitude to within 4 cm at every station and gives a friction coefficient of $r = (2.1 \pm 0.3) \times 10^{-3} \text{ m s}^{-1}$. The damping length of 38 km is consistent with the depth and width convergence of the channel.

1. Introduction

Tides entering a converging estuary are shaped by two competing effects: the narrowing of the channel, which amplifies the wave, and bottom friction, which damps it¹. Which effect wins is set by the ratio of the convergence length to the frictional length, and both are measurable from a handful of tide gauges.

2. Model

For a channel of depth h and width $b(x) = b_0 e^{-x/L_b}$, the linearized shallow-water equations for the tidal elevation η and velocity u are

$$\frac{\partial \eta}{\partial t} + \frac{1}{b} \frac{\partial (bhu)}{\partial x} = 0, \quad \frac{\partial u}{\partial t} + g \frac{\partial \eta}{\partial x} + \frac{r}{h} u = 0$$

where r is the linearized friction coefficient. Seeking solutions of the form $\eta = \hat{\eta}(x)e^{i\omega t}$ gives a damped wave

$$\hat{\eta}(x) = \eta_0 \exp\left(-\frac{x}{L_d}\right) \exp(-ikx), \quad \frac{1}{L_d} = \frac{1}{2L_b} - \text{Re}(\kappa), \quad \kappa = \frac{\omega}{\sqrt{gh}} \sqrt{1 - \frac{ir}{\omega h}}$$

so the amplitude ratio between two stations a distance Δx apart is $\exp(-\Delta x / L_d)$. The solution has one free parameter, r , once h and L_b are read from the bathymetry.

3. Data

Water levels were recorded every 10 minutes for 35 days, from 1 March to 4 April 2026, at the six stations in Table 1. The semidiurnal amplitude A_{M_2} was obtained by harmonic analysis of each record.

Table 1. Stations and observed semidiurnal amplitude.

Station	Distance from mouth (km)	Depth (m)	Width (m)	A_{M_2} (m)
S1, mouth	0.0	9.2	2,400	1.42 ± 0.02

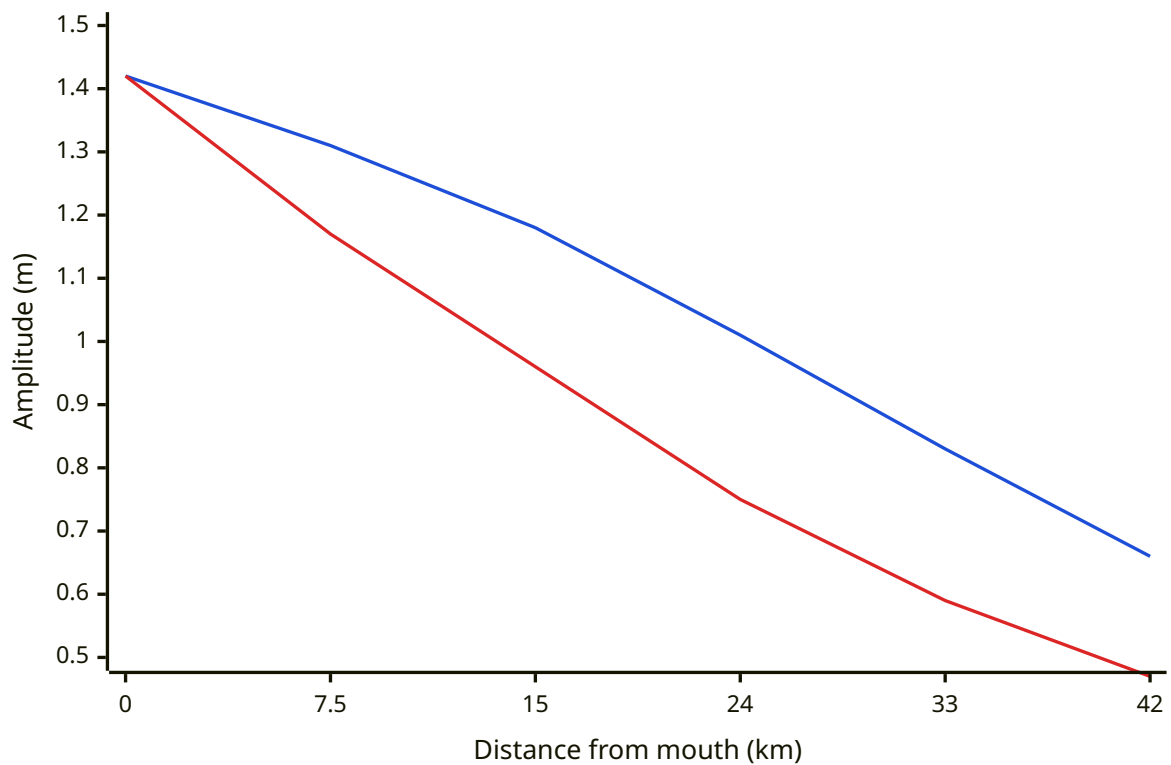
¹ Savenije, H. H. G. (2012). *Salinity and Tides in Alluvial Estuaries*, 2nd edition. Available from the author.

Station	Distance from mouth (km)	Depth (m)	Width (m)	A_{M_2} (m)
S2	7.5	8.1	1,650	1.31 ± 0.02
S3	15.0	7.4	1,100	1.18 ± 0.02
S4	24.0	6.0	720	1.01 ± 0.03
S5	33.0	5.2	450	0.83 ± 0.03
S6, weir	42.0	4.6	300	0.66 ± 0.03

4. Results

A least-squares fit of the damped wave to the six amplitudes gives $r = (2.1 \pm 0.3) \times 10^{-3} \text{ m s}^{-1}$ and $L_d = 38 \pm 3 \text{ km}$. Figure 1 compares the fitted curve with the observations.

Figure 1. Semidiurnal amplitude along the estuary



The blue line is the observed amplitude, the red line the amplitude a channel of constant width would show with the same friction. The difference is the amplification by convergence, which offsets about half of the frictional loss.

The residuals are listed in Table 2. The largest, at S4, coincides with a side channel that the one-dimensional model does not represent.

Table 2. Fitted against observed amplitude.

Station	Observed (m)	Fitted (m)	Residual (cm)
S1	1.42	1.42	0
S2	1.31	1.29	+2
S3	1.18	1.17	+1

Station	Observed (m)	Fitted (m)	Residual (cm)
S4	1.01	1.05	-4
S5	0.83	0.85	-2
S6	0.66	0.67	-1

5. Discussion

The fitted friction coefficient agrees with the value $r \approx \frac{8}{3\pi} C_d U$ expected for a drag coefficient $C_d = 2.5 \times 10^{-3}$ and a velocity amplitude $U \approx 1.0 \text{ m s}^{-1}$, which gives $2.1 \times 10^{-3} \text{ m s}^{-1}$. The model therefore needs no tuning beyond the measured bathymetry².

Two limits of the approach should be stated. The linearization assumes $\eta \ll h$, which holds at the mouth ($\eta / h = 0.15$) but is marginal at the weir (0.14 with a depth of 4.6 m at the lowest water). And the model ignores river discharge, which was below $40 \text{ m}^3 \text{ s}^{-1}$ throughout the record.

6. Conclusion

Six tide gauges and a bathymetric chart are enough to fit a one-dimensional tidal model to a converging estuary with a residual of a few centimeters. The friction coefficient is physically reasonable and the damping length, 38 km, is a property of the channel that can be compared across estuaries.

Acknowledgements

The port authority provided the water-level records. The bathymetry is from the 2024 hydrographic survey.

All organizations and data in the sample documents are invented. Names and figures are illustrative.

² Friedrichs, C. T. and Aubrey, D. G. (1994). Tidal propagation in strongly convergent channels. *Journal of Geophysical Research*, 99(C2), 3321 to 3336.