

Analysis and number theory

Limits and continuity

$$\lim_{x \rightarrow a} f(x) = L \iff \forall \varepsilon > 0 \exists \delta > 0: 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \lim_{n \rightarrow \infty} \sqrt[n]{n} = 1 \quad \liminf_{n \rightarrow \infty} a_n \leq \limsup_{n \rightarrow \infty} a_n$$

Series

Taylor's theorem with the Lagrange remainder:

$$f(x) = \sum_{k=0}^n \frac{f^{(k)}(a)}{k!} (x-a)^k + \frac{f^{(n+1)}(\xi)}{(n+1)!} (x-a)^{n+1}, \quad \xi \text{ between } a \text{ and } x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad (|x| < 1)$$

A Fourier series and the transform pair:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos \frac{2\pi n t}{T} + b_n \sin \frac{2\pi n t}{T} \right), \quad a_n = \frac{2}{T} \int_0^T f(t) \cos \frac{2\pi n t}{T} dt$$

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx \quad f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{2\pi i x \xi} d\xi \quad F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

Complex analysis

$$f(z) = \frac{1}{2\pi i} \oint_{\gamma} \frac{f(w)}{w-z} dw \quad \oint_{\gamma} f(z) dz = 2\pi i \sum_k \text{Res}(f, z_k) \quad e^{i\theta} = \cos \theta + i \sin \theta$$

Number theory

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1-p^{-s}} \quad \pi(x) \sim \frac{x}{\ln x} \quad \sum_{d|n} \varphi(d) = n$$

$$a^{p-1} \equiv 1 \pmod{p} \quad (p \nmid a) \quad \gcd(a, b) \cdot \text{lcm}(a, b) = ab \quad \binom{p}{k} \equiv 0 \pmod{p} \quad (0 < k < p)$$

Stirling's approximation and Ramanujan's series for $1/\pi$:

$$n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \quad \frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)! (1103 + 26390k)}{(k!)^4 396^{4k}}$$

The Gamma and Beta functions:

$$\Gamma(z) = \int_0^{\infty} t^{z-1} e^{-t} dt, \quad \Gamma(n) = (n-1)!, \quad B(x, y) = \frac{\Gamma(x) \Gamma(y)}{\Gamma(x+y)}$$

Linear algebra

Eigenvalues, the spectral theorem and the singular value decomposition:

$$A\mathbf{v} = \lambda\mathbf{v} \quad \det(A - \lambda I) = 0 \quad A = Q\Lambda Q^\top \quad (A = A^\top) \quad A = U\Sigma V^\top$$

$$\|\mathbf{x}\|_p = \left(\sum_{i=1}^n |x_i|^p \right)^{1/p} \quad \langle \mathbf{u}, \mathbf{v} \rangle = \sum_i u_i \overline{v_i} \quad \text{tr}(AB) = \text{tr}(BA)$$