

Engineering and finance formulas

Signals and control

A first-order low-pass filter, its cut-off frequency and the sampling theorem:

$$H(s) = \frac{1}{1 + sRC} \quad f_c = \frac{1}{2\pi RC} \quad f_s > 2f_{\max}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \quad \text{gain in dB} = 20 \log_{10} |H(j\omega)| \quad y[n] = \sum_{k=0}^M b_k x[n-k] - \sum_{k=1}^N a_k y[n-k]$$

The PID controller and the discrete Fourier transform:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \quad X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi i k n / N}$$

Electrical

$$V = IR \quad P = VI = I^2 R = \frac{V^2}{R} \quad \sum_k I_k = 0 \quad Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Mechanical and civil

Deflection of a simply supported beam under a central load, stress and strain, and the Reynolds number:

$$\delta_{\max} = \frac{FL^3}{48EI} \quad \sigma = \frac{F}{A} = E\varepsilon \quad I_{\text{rect}} = \frac{bh^3}{12} \quad \text{Re} = \frac{\rho v L}{\mu}$$

$$Q = Av \quad p_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2 \quad \Delta p = f \frac{L}{D} \frac{\rho v^2}{2}$$

Interest and loans

Compound interest, continuous compounding, and the payment on an annuity loan of principal P at periodic rate r over n periods:

$$A = P\left(1 + \frac{r}{m}\right)^{mt} \quad A = Pe^{rt} \quad \text{payment} = P \frac{r(1+r)^n}{(1+r)^n - 1}$$

A loan of $P = 250,000$ at 0.35% a month over $n = 300$ months costs

$$250,000 \times \frac{0.0035 \cdot 1.0035^{300}}{1.0035^{300} - 1} \approx 1,518 \text{ a month.}$$

Investment appraisal

$$\text{NPV} = \sum_{t=0}^T \frac{C_t}{(1+r)^t} \quad \text{IRR: } \sum_{t=0}^T \frac{C_t}{(1+\text{IRR})^t} = 0 \quad \text{WACC} = \frac{E}{V} r_E + \frac{D}{V} r_D (1 - \tau)$$

The capital asset pricing model and the Sharpe ratio:

$$\mathbb{E}[R_i] = R_f + \beta_i (\mathbb{E}[R_m] - R_f) \quad \beta_i = \frac{\text{Cov}(R_i, R_m)}{\text{Var}(R_m)} \quad S = \frac{\mathbb{E}[R] - R_f}{\sigma}$$

Option pricing

The Black-Scholes partial differential equation and the price of a European call:

$$\frac{\partial V}{\partial t} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} - rV = 0$$

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2), \quad d_1 = \frac{\ln(S_0 / K) + \left(r + \frac{1}{2} \sigma^2\right) T}{\sigma \sqrt{T}}, \quad d_2 = d_1 - \sigma \sqrt{T}$$

Greek	Definition	Call
Delta	$\frac{\partial C}{\partial S}$	$N(d_1)$
Gamma	$\frac{\partial^2 C}{\partial S^2}$	$\frac{N'(d_1)}{S_0 \sigma \sqrt{T}}$
Theta	$\frac{\partial C}{\partial t}$	$-\frac{S_0 N'(d_1) \sigma}{2\sqrt{T}} - rKe^{-rT} N(d_2)$
Vega	$\frac{\partial C}{\partial \sigma}$	$S_0 \sqrt{T} N'(d_1)$