

Equations of physics

Classical mechanics

Newton's second law and the gravitational force, then the Lagrangian form:

$$\mathbf{F} = m \ddot{\mathbf{r}} \quad \mathbf{F} = -\frac{GMm}{r^2} \hat{\mathbf{r}} \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = 0$$

A simple pendulum of length ℓ has period $T = 2\pi\sqrt{\ell/g}$ for small angles.

Electromagnetism

Maxwell's equations in differential form:

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{B} &= \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}\end{aligned}$$

The same in integral form, and the Lorentz force:

$$\oint_{\partial S} \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d}{dt} \iint_S \mathbf{B} \cdot d\mathbf{A} \quad \mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Quantum mechanics

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t) \right] \Psi(\mathbf{r}, t)$$

$$(i\gamma^\mu \partial_\mu - m) \psi = 0 \quad [\hat{x}, \hat{p}] = i\hbar \quad \Delta x \Delta p \geq \frac{\hbar}{2} \quad \langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$$

The hydrogen energy levels and the Planck relation:

$$E_n = -\frac{m_e e^4}{8\epsilon_0^2 \hbar^2} \cdot \frac{1}{n^2} \approx -\frac{13.6 \text{ eV}}{n^2} \quad E = h\nu = \hbar\omega$$

Relativity

A boost along x with $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2}$:

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad E^2 = (pc)^2 + (mc^2)^2$$

Fluids and heat

Navier-Stokes for an incompressible fluid, the heat equation, and Planck’s law of black-body radiation:

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g}, \qquad \nabla \cdot \mathbf{v} = 0$$

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u \qquad B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/k_B T} - 1} \qquad S = k_B \ln \Omega$$

Constant	Symbol	Value
Speed of light	c	$2.998 \times 10^8 \text{ m s}^{-1}$
Planck constant	h	$6.626 \times 10^{-34} \text{ J s}$
Gravitational constant	G	$6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
Boltzmann constant	k_B	$1.381 \times 10^{-23} \text{ J K}^{-1}$